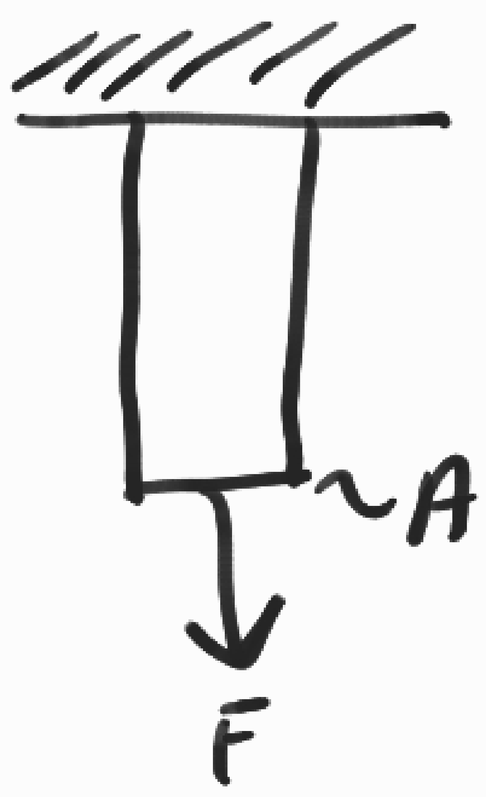
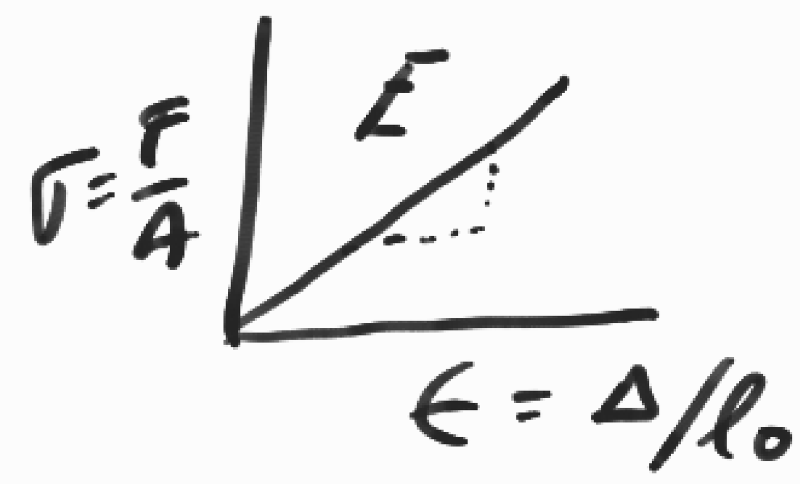
5. CONSTITUTIVE LAWS

THESE ARE THE EQUATIONS THAT DEFINE HOW STRESS IS RELATED TO STRAIN. FOR EXAMPLE (4.3)





$$\boxed{\sigma = E\epsilon}$$

is a constit. law

INCOMPRESSIBLE - A MATERIAL THAT DOES NOT CHANGE VOLUME

COMPRESSIBLE - CAN CHANGE VOLUME (SPONGE)

ELASTIC - A DEFORMATION THAT IS FULLY REVERSED WHEN FORCES ARE REMOVED (RUBBER BAND SNAPS BACK)

5.1 LINEAR ELASTIC:

STRESS $\underset{\approx}{\sigma}$ IS LINEARLY RELATE TO $\underset{\approx}{\epsilon}$, AS ABOVE. BUT IF WE DO NOT ASSUME 1D, OR ISOTROPIC, 4.3 GENERALIZES TO

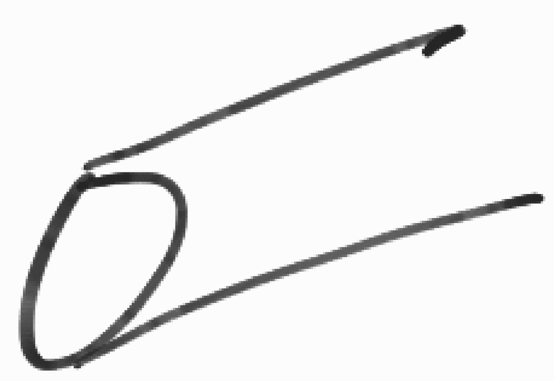
$$\underset{\approx}{\sigma} = \underset{\approx\approx}{C}\,\underset{\approx}{\epsilon} \qquad (5.1)$$

IN COMPONENT FORM

$$\sigma_{ij} = C_{ijkl}\,\epsilon_{kl} \qquad (5.2)$$

WHERE $\underset{\approx\approx}{C}$ IS THE 4TH ORDER STIFFNESS TENSOR.

ISOTROPIC - ISOTROPIC LINEAR ELASTIC SOLIDS ARE DEFINED BY 2 PARAMETERS ONLY



eg fat, liver

E - Youngs Modulus

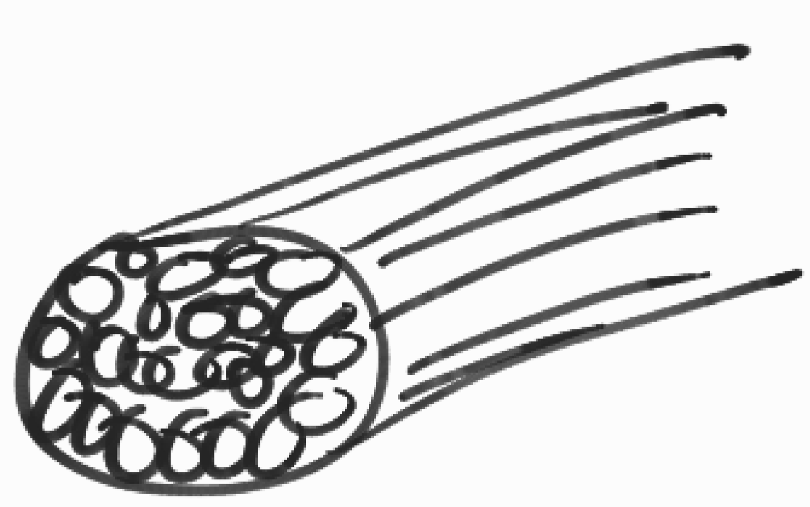
ν - Poisson's Ratio

$\nu = -\varepsilon_t / \varepsilon_a$ (transverse / applied)

$$\sigma_{ij} = \frac{E}{(1+\nu)}\left[\epsilon_{ij} + \frac{\nu}{1-2\nu}\epsilon_{kk}\delta_{ij}\right] \qquad 5.4$$

FOR INCOMPRESSIBLE MATERIALS $\nu = 0.5$

TRANSVERSELY ISOTROPIC - ISOTROPIC EXCEPT IN ONE DIRECTION. EXAMPLES: tendon, skeletal muscle

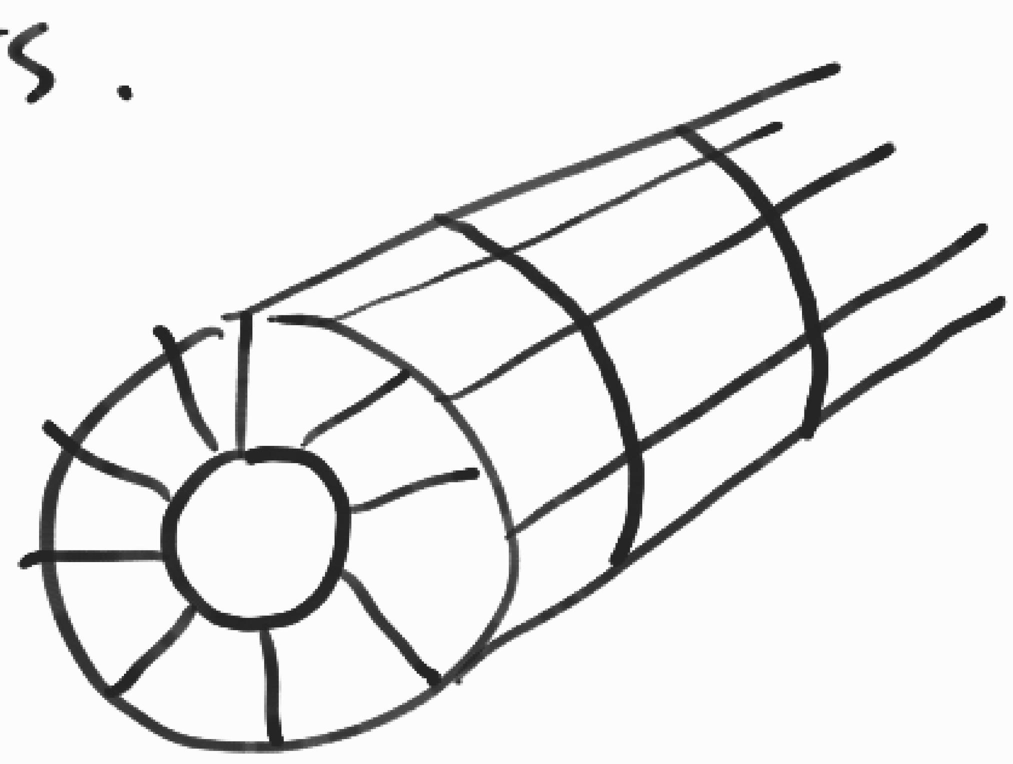


DEFINED BY 5 CONSTANTS

$E_t, \nu_t, E_f, \nu_f, \nu_{tf}$

ORTHOTROPIC - PROPERTIES DIFFER ALONG 3 ORTHOGONAL AXES.

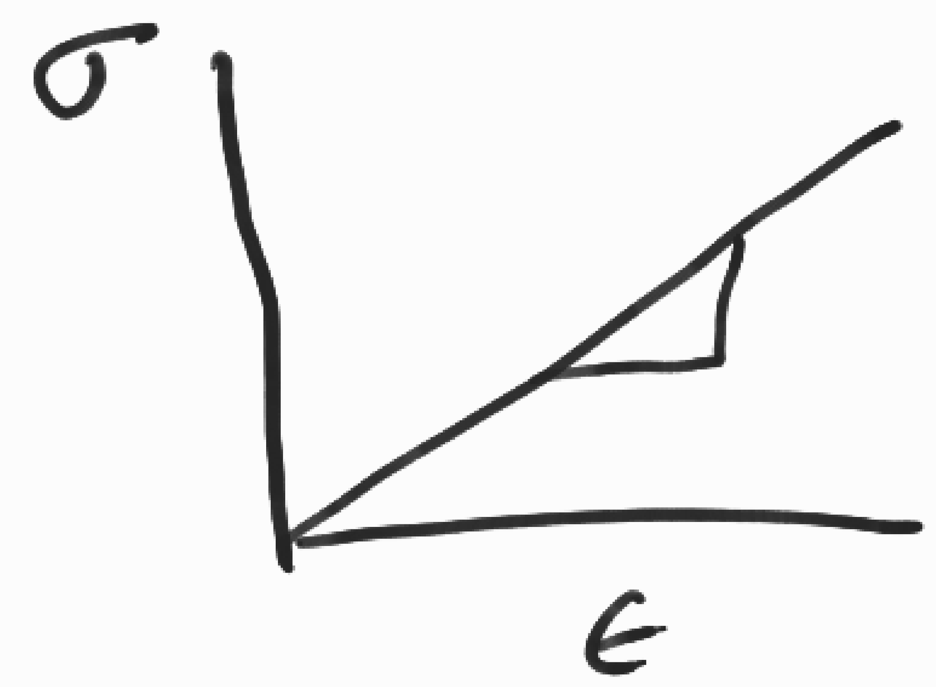
EXAMPLE: ARTERIES



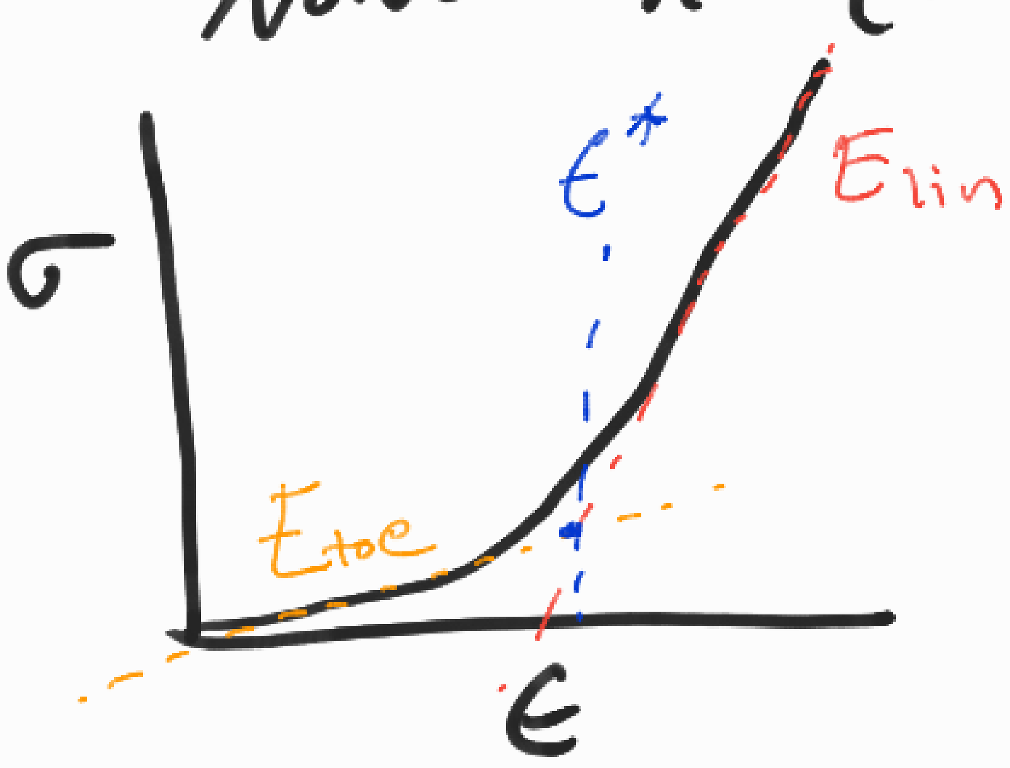
DEFINED BY 9 CONSTANTS.

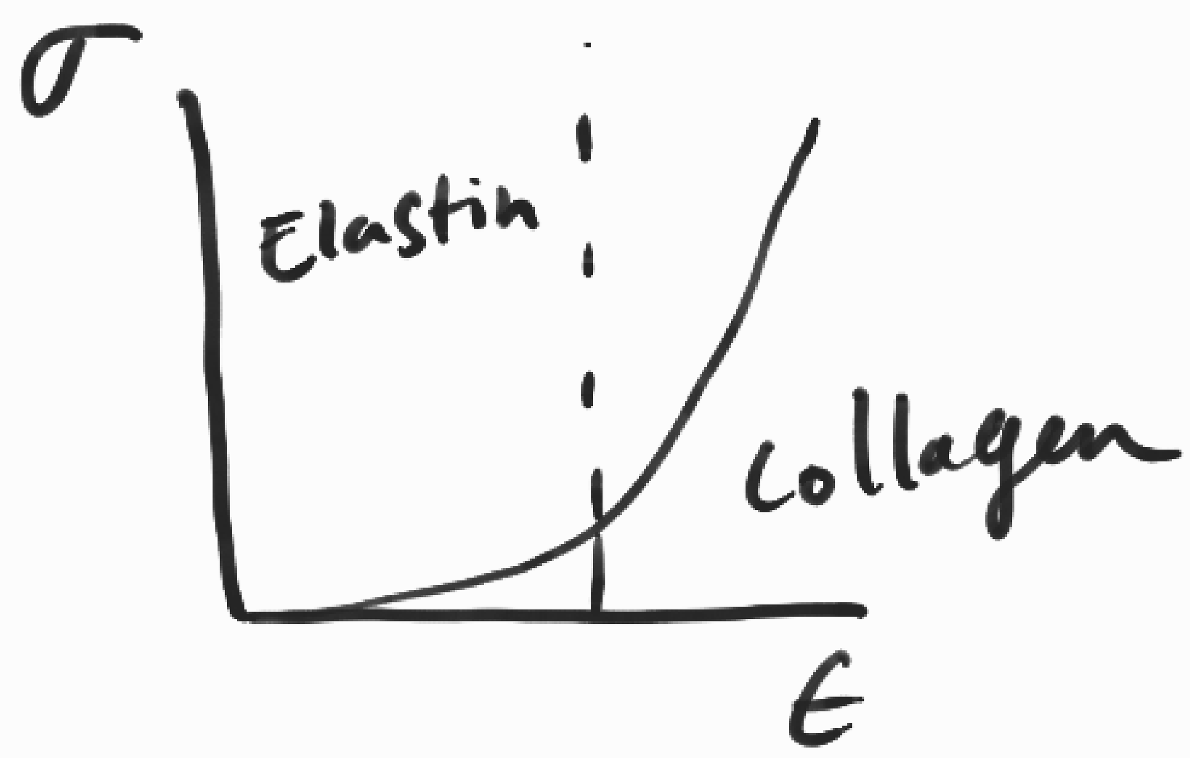
5.2 Nonlinerity

LINEAR (BONE)



NONLINEAR (TENDON)





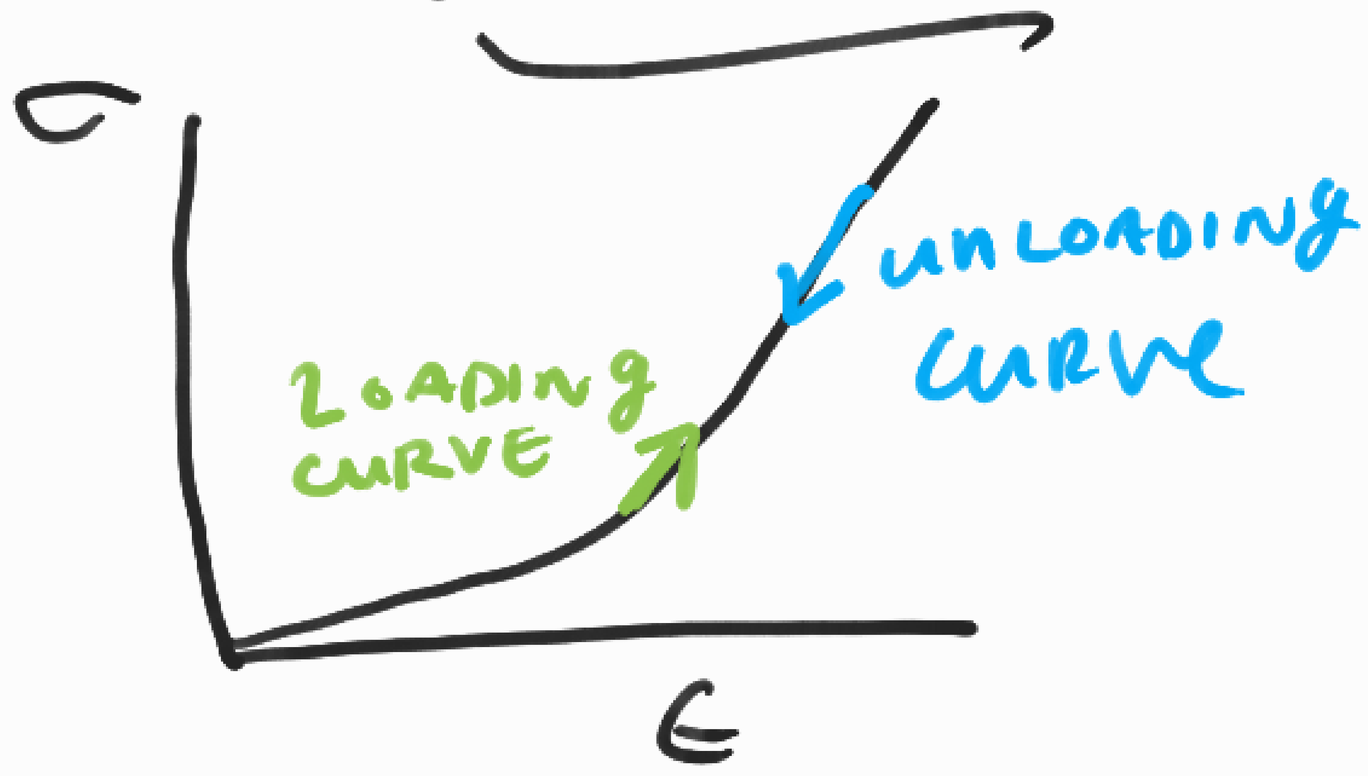
E_{toe} - Toe region Modulus

E_{lin} - linear region Modulus

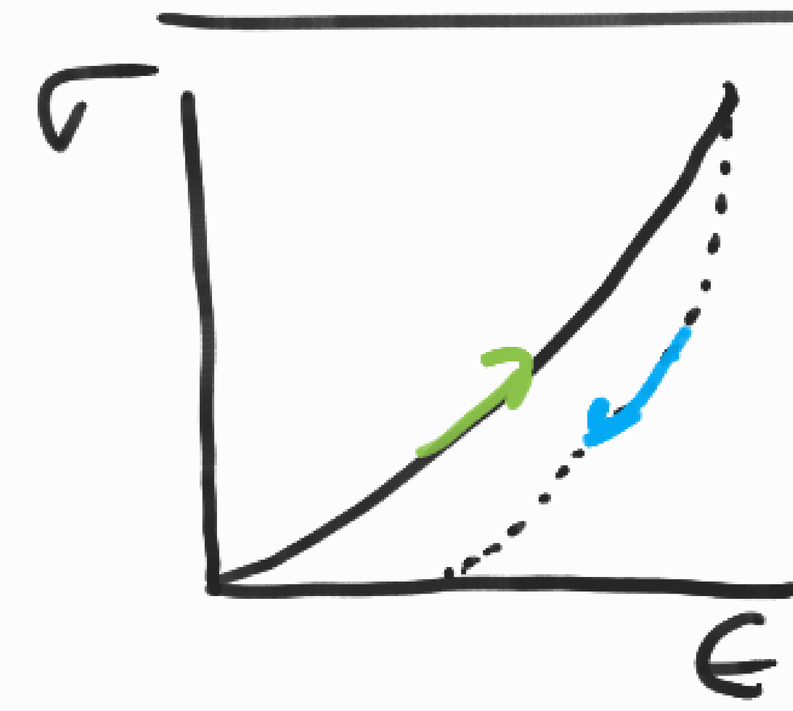
ϵ^* - transition strain

5.3 Viscoelasticity -

ELASTIC

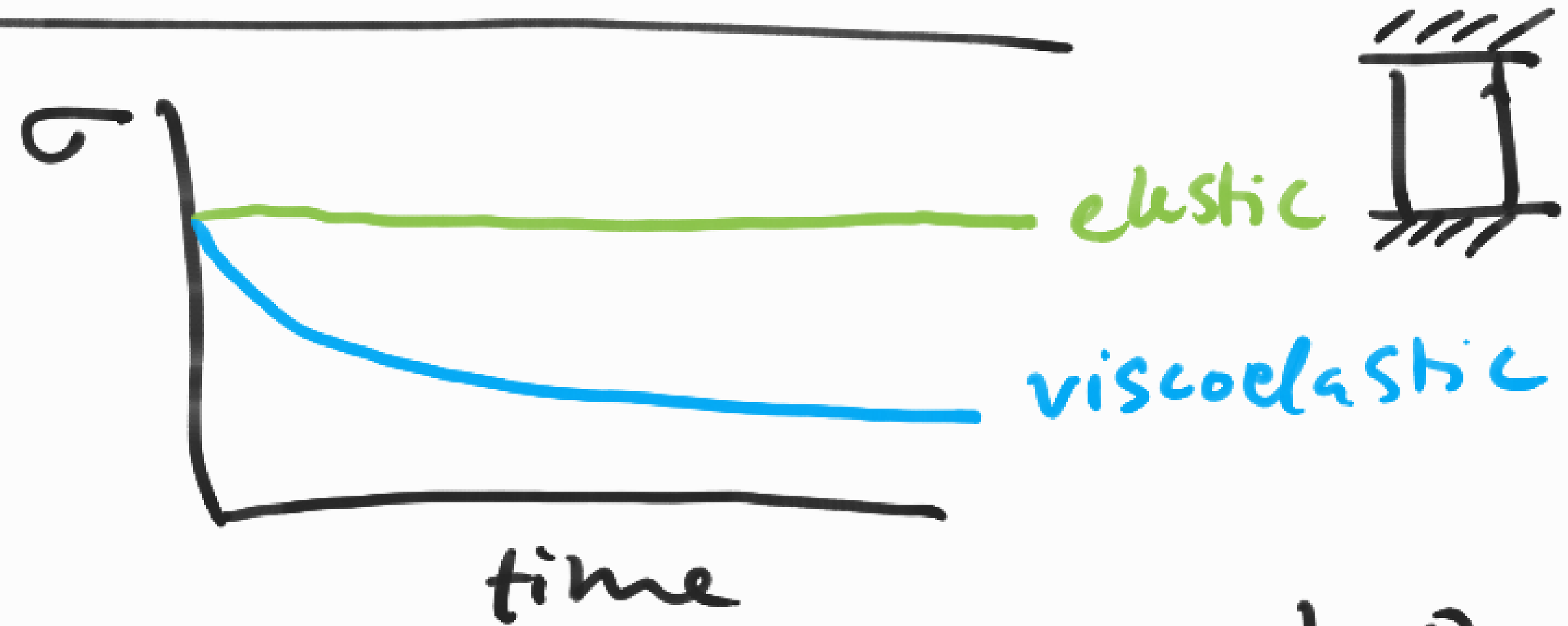


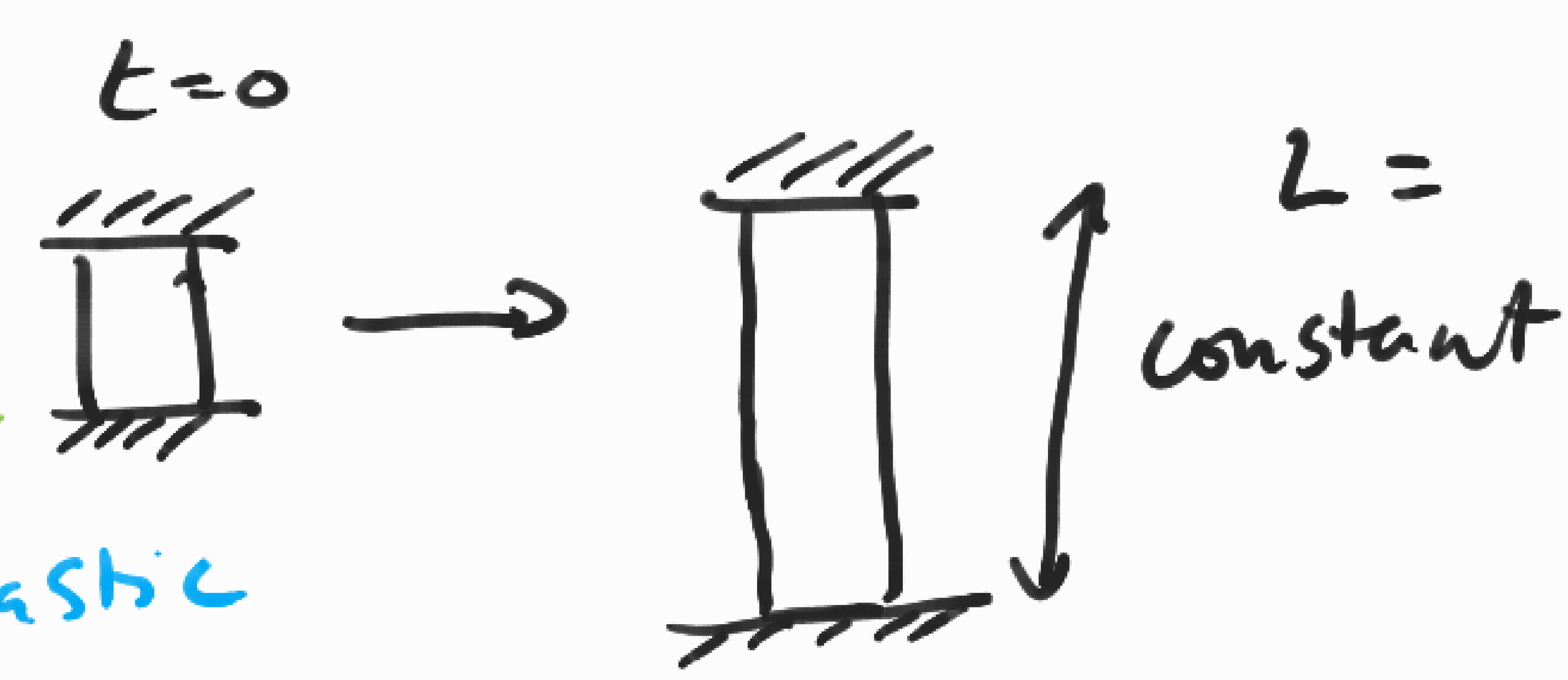
VISCOELASTIC



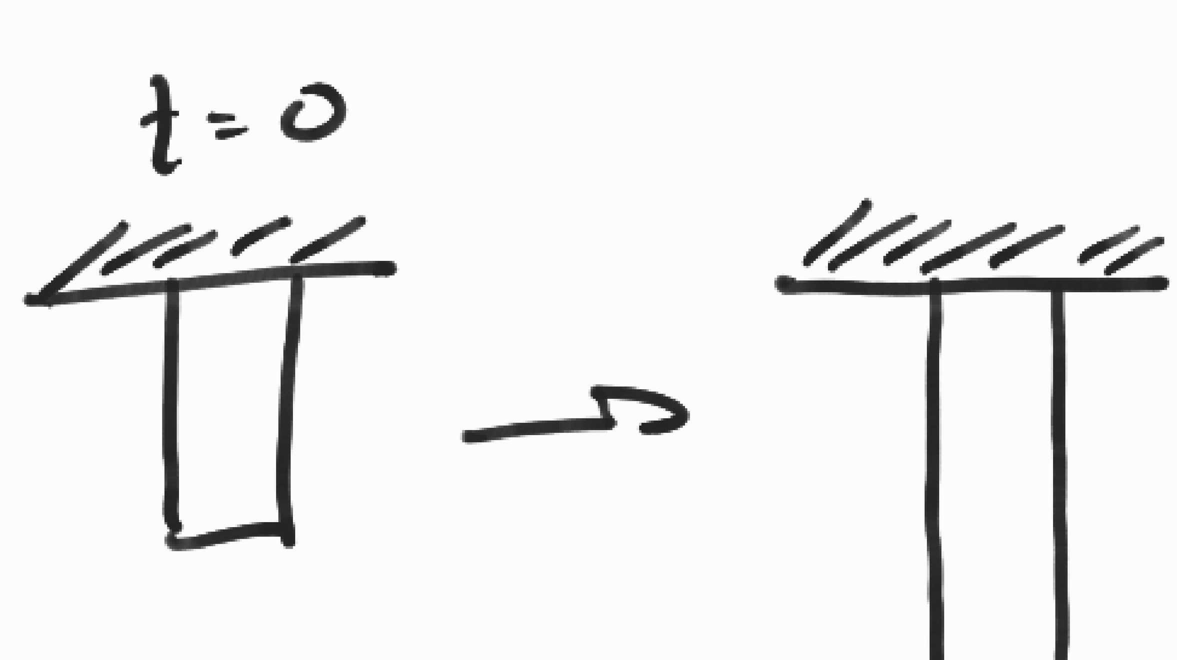
HYSTERESIS LOOP

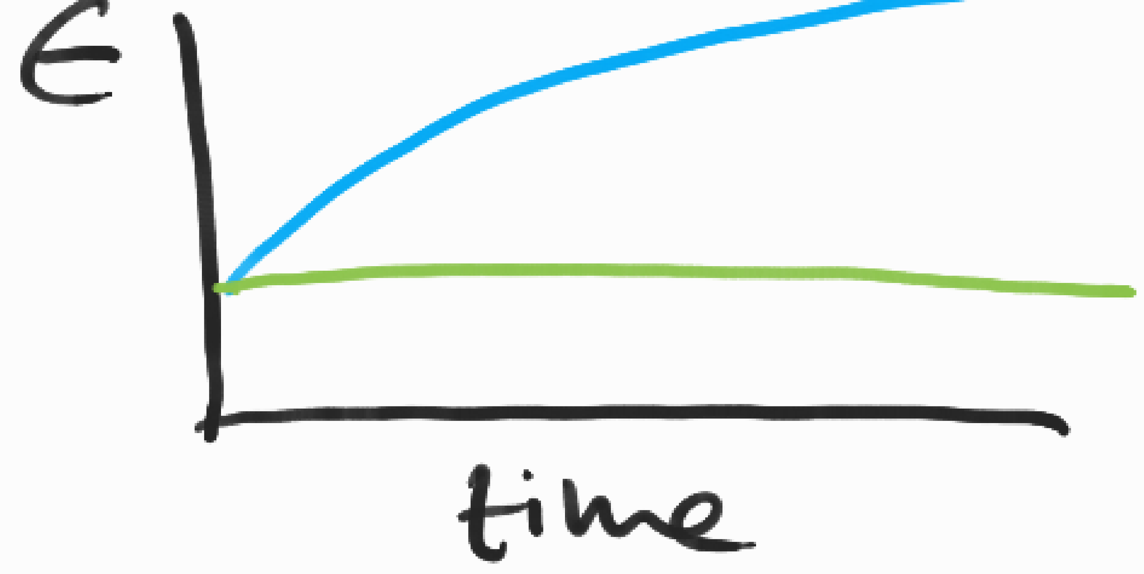
STRESS RELAXATION

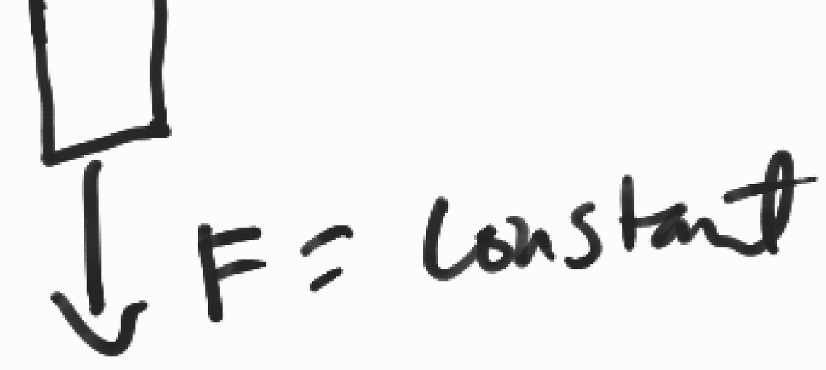




CREEP




ε
time

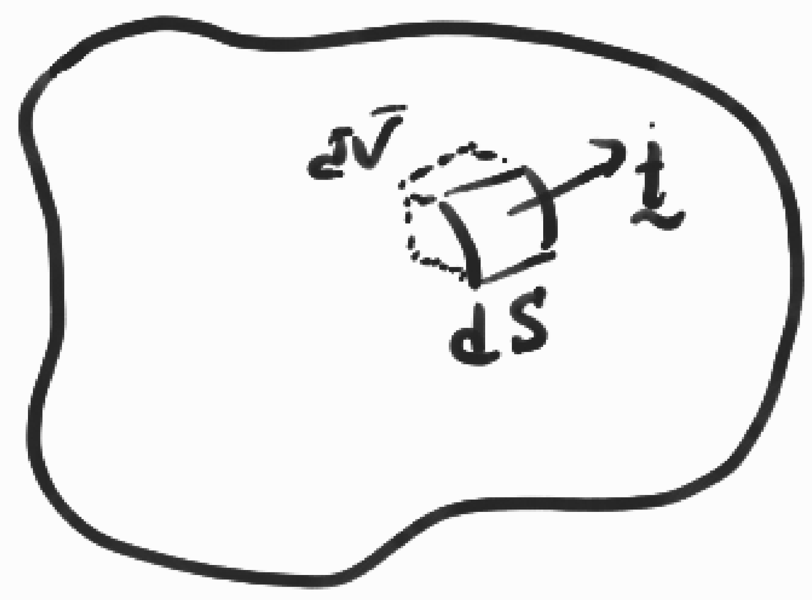

F = constant

6. EQUATIONS OF MOTION

CONSIDER A BODY WITH SURFACE S AND VOLUME V

SUBJECT TO FORCES ON ITS SURFACE $\underset{\sim}{t}^{(n)}$ $n = 1, 2, 3, 4 \ldots$

AND BODY FORCES $\underset{\sim}{f}$



WE BEGIN WITH

$$\Sigma \underset{\sim}{F} = m \underset{\sim}{a} = \frac{d}{dt}(m \underset{\sim}{v}) \qquad 6.1$$

WHICH CAN BE REWRITTEN AS

$$\int_S \underset{\sim}{n} \cdot \underset{\approx}{\sigma} \, dS + \int_V f \, dV = \frac{d}{dt} \int_V \underset{\sim}{v} \rho \, dV \qquad 6.2$$

DIVERGENCE THEOROM SAYS

$$\int_S \underset{\sim}{n} \cdot \underset{\approx}{F} \, dS = \int_V \underset{\sim}{\nabla} \cdot \underset{\approx}{F} \, dV \qquad 6.3$$

CAN USE 6.3 TO CONVERT 6.2 TO ALL VOLUME INTEGRALS

(Leibnitz Rule)

$$\int \underset{\sim}{\nabla} \cdot \underset{\approx}{\sigma} \, dV + \int f \, dV = \int \frac{\partial}{\partial t} (\underset{\sim}{v} \rho \, dV) \qquad 6.4$$

$$= \int_V \frac{d\underset{\sim}{v}}{dt}\,\rho\, dV + \int_V \underset{\sim}{v}\, \underbrace{\frac{\partial}{\partial t}(\rho\, dV)}_{0}$$

constant mass

hence

$$\int_V \underset{\sim}{\nabla} \cdot \underset{\approx}{\sigma}\, dV + \int_V \underset{\sim}{f}\, dV = \int_V \rho\, \underset{\sim}{a}\, dV$$

or

$$\boxed{\underset{\sim}{\nabla} \cdot \underset{\approx}{\sigma} + \underset{\sim}{f} = \rho\, \underset{\approx}{a}} \qquad 6.5$$

"EQUATIONS OF MOTION"

IN CARTESIAN COMPONENTS,

$$\boxed{\frac{\partial \sigma_{ij}}{\partial x_i} + f_j = \rho\, a_j} \qquad 6.6$$